Stars and their variability observed from space
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THE COMPLEX ASTEROSEISMOLOGY OF SX PHOENICIS

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Abstract. This paper presented a seismic analysis of the prototype SX Phoenicis, and was aimed at fitting the two radial-mode frequencies and the corresponding values of the bolometric flux amplitude (the parameter f), whose empirical values were derived from multi-coulor photometric observations. A seismic model that meets those conditions is of low mass ($M = 1.05M_{\odot}$), has moderately effective convection in the outer layers and described by the mixing-length parameter $\alpha_{\rm MLT} \approx 0.7$, and a microturbulent velocity in the atmospheres of about $\xi_{\rm t} \approx 8$ km s^{-1}. These seismic studies of a star like SX Phe are very important for deriving constraints on outer-layer convection because the object is borderline between very effective and ineffective convection.

Keywords: Stars: evolution, atmospheres, asteroseismology, convection

1 Introduction

SX Phoenicis (HD 223065) is A3-type star of Population II discovered to be variable almost seven decades ago by Eggen (1952a,b). This is a prototype for the whole class of high-amplitude and usually metal-poor pulsators located inside the δ Scuti instability region. SX Phe was a target of several studies based on photometric and spectroscopic observations. Analyses of photometric data revealed two frequencies and their combinations (e.g., by Coates et al. 1979; Rolland et al. 1991; Garrido & Rodriguez 1996). The frequency ratio indicated that SX Phe pulsates in two radial modes: fundamental and first overtone ones. These two periodicities are also present in the radial-velocity variations (Kim et al. 1993). Interestingly, the recent analysis of high-precision data from *TESS* confirmed the former results that these two frequencies alone dominate the light-curve of SX Phe (Antoci et al. 2019). The frequencies extracted from the *TESS* light-curve were $\nu_1 = 18.193565(6)$ d^{-1} and $\nu_2 = 23.37928(2)$ d^{-1}.

There was also some evidence that both pulsation periods change on a time-scale of decades (Landes et al. 2007). Moreover, for the dominant pulsational period the effective temperature varies in a huge range (7210–8120 K) and the surface gravity from 3.63–4.23. The corresponding mean values are 7640 K and 3.89 (Kim et al. 1993). A recent determination of the effective temperature from spectroscopy gives $T_{\rm eff} = 7500 \pm 150$ K and the luminosity derived from the Gaia DR2 data is $\log L/L_{\odot} = 0.844 \pm 0.009$ (Antoci et al. 2019). The metallicity of SX Phe is low, and amounts to about [Fe/H] = –1.4 (McNamara 1997).

SX Phe has been also a subject of seismic modelling. In the first attempts, Dziembowski & Kozłowski (1974) derived a small mass of about $0.2\,M_{\odot}$. However, modelling by Petersen & Christensen-Dalsgaard (1996) with the recomputed OPAL opacity data (Iglesias et al. 1992) showed that the period ratio of the two radial modes is best reproduced by a model with parameters mass $M = 1.0M_{\odot}$, metallicity $Z = 0.001$, initial hydrogen abundance $X_0 = 0.70$ and age = 4.07 Gyr.

The aim of this paper is to repeat the seismic modelling of SX Phe and to extend it by employing the bolometric flux amplitude (the parameter f), which is very sensitive to physical conditions in sub-photospheric layers.

2 Fitting models to the two radial-mode frequencies

Using the Warsaw–New Jersey evolutionary code (e.g., Pamyatnykh et al. 1998; Pamyatnykh 1999) and a non-adiabatic pulsational code (Dziembowski 1977), we computed models of SX Phe with the specific aim of fitting ν_1 and ν_2 as the radial fundamental and first overtone modes, respectively. Part of these results have been already published in Antoci et al. (2019).

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We searched the whole range of effective temperatures for SX Phe reported in the literature, i.e., from about 7000–9000 K. The OPAL opacity tables and the solar abundance mixture of Asplund et al. (2009) were adopted. We considered different values of metallicity (Z) and initial hydrogen abundance (X_0), from the range (0.0005, 0.003) and (0.66, 0.74), respectively. Different values of the mixing-length padrameter were investigated: $\alpha_{\mathrm{MLT}} \in (0,\ 2.5)$; overshooting from the convective core was not included. At such low metallicity and effective temperatures $T_{\mathrm{eff}} > 7000\,\mathrm{K}$, the effect of α_{MLT} on mode frequencies would be negligible, but it strongly affects the values of the parameter f, as discussed in the next section.

We started by re-calculating the seismic model of Petersen & Christensen-Dalsgaard (1996), which had constraints of mass $M = 1.0\ M_{\odot}$, metallicity $Z = 0.001$ and initial hydrogen $X_0 = 0.70$. Our best seismic model with those parameters had an effective temperature $\log T_{\mathrm{eff}} = 3.88427$, luminosity $\log L/L_{\odot} = 0.811$, and a frequency ratio (of the radial fundamental mode to the first overtone) of $\nu_1/\nu_2 = 0.780014$. The observed counterpart was 0.778192. The theoretical values of the frequencies were $\nu_1 = 18.193565\ \mathrm{d}^{-1}$ (the model was interpolated at that frequency) and $\nu_2 = 23.324654\ \mathrm{d}^{-1}$. Both radial modes in this model were excited. However, as one can see, the difference between the theoretical and observed value of ν_1/ν_2 is significant, and amounted to 0.001822. The age given by the model was about 3.93 Gyr, which is slightly less than for the model of Petersen & Christensen-Dalsgaard (1996), who obtained about 4.07 Gyr, and may result from different versions of the OPAL opacity tables and from using slightly differentr evolutionary and pulsational codes.

The next step was to change the value of the metallicity, and we found that the model for $Z = 0.0014$ with a mass $M = 1.15\ M_{\odot}$ fitted the observed frequency ratio better. It had the following parameters: $\log T_{\mathrm{eff}} = 3.91770$, luminosity $\log L/L_{\odot} = 0.984$, and a frequency ratio of $\nu_1/\nu_2 = 0.778412$; individual frequencies were $\nu_1 = 18.193565\ \mathrm{d}^{-1}$ and $\nu_2 = 23.372681\ \mathrm{d}^{-1}$. It was also much younger than the previous model we derived, having an age of about 2.45 Gyr. The fundamental and first overtone radial modes were both stable. Increasing the metallicity slightly, to $Z = 0.002$, enabled us to find a model with a mass of $M = 1.05\ M_{\odot}$, $\log T_{\mathrm{eff}} = 3.87152$, $\log L/L_{\odot} = 0.770$, and a frequency ratio matching the two radial modes considered: $\nu_1/\nu_2 = 0.778073$. This models was about 3.42 Gyr old; both radial fundamental and first overtone modes were excited. As we could see, the models with a higher metallicity gave much better fit to the frequencies, but there was still room for improvement, so in the next step we changed the initial abundance of hydrogen.

We managed to find a model that reproduced the observed frequency ratio up to the fifth place of decimals (which we adopted as the numerical accuracy). It had the following parameters: $M = 1.05\ M_{\odot}$, $X_0 = 0.67$, $Z = 0.002$, $\log T_{\mathrm{eff}} = 3.88979$, $\log L/L_{\odot} = 0.844$; it was interpolated at the dominant frequency $\nu_1 = 18.193565\ \mathrm{d}^{-1}$. The value of the second frequency was $\nu_2 = 23.379761\ \mathrm{d}^{-1}$, which differed by $0.00048\ \mathrm{d}^{-1}$ from the observed value. The theoretical value of the frequency ratio was therefore 0.778176, compared to the observed value 0.778192. Taking into account the numerical accuracy, which was not better than five decimal places, we could conclude that this model reproduced perfectly the observed frequencies of the two radial modes of SX Phe. Moreover, it predicted instability (excitation) of both the radial fundamental and the first overtone modes. It gave an age of about 2.85 Gyr. Another advantage of the model was the fact that it had a luminosity which agreed with the value derived from the *Gaia* DR2 data.

The lower hydrogen abundance (higher helium abundance) was also quite probable because, like other SX Phoenicis variables, SX Phe itself can be a blue straggler. Such objects are presumably formed by the merger of two stars, or by interactions in a binary system and as a consequence they may have enhanced helium abundances (e.g., McNamara 2011; Nemec et al. 2017).

The left panel of Fig. 1 shows the evolution of the frequency ratio as a function of the dominant frequency (the "Petersen diagram") for the 4 best seismic models that we derived. The observed value is marked as a solid square. The right panel of Fig. 1 depicts the corresponding evolutionary tracks on the H–R diagram and the position of SX Phe.

3 Complex seismic modeling

The next step was to extend the seismic analysis by including the parameter which describes the relative amplitude of the radiative flux perturbations at the photosphere level. This amplitude is the so-called nonadiabatic parameter f, defined by:

$$\frac{\delta \mathcal{F}_{\mathrm{bol}}}{\mathcal{F}_{\mathrm{bol}}} = \mathrm{Re}\{\varepsilon f Y_{\ell}^{m}(\theta,\varphi)\mathrm{e}^{-\mathrm{i}\omega t}\}, \tag{3.1}$$

where $\mathcal{F}_{\mathrm{bol}}$ is the bolometric flux, ε is the mode intrinsic amplitude, Y_{ℓ}^{m} is the spherical harmonic with degree ℓ and azimuthal order m. Other quantities have their usual meanings. The value of f is complex and is associated with a given mode. The theoretical values of f are very sensitive to the subphotospheric layers from which

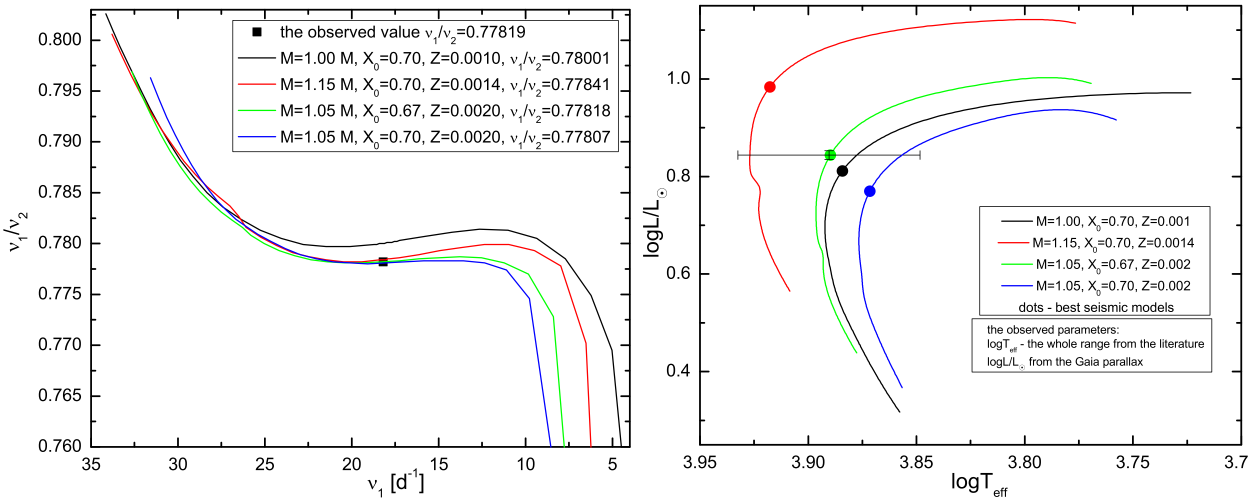

Fig. 1. Left: Petersen diagram for the 4 best seismic models suitable for SX Phoenicis. The solid square marks the observed value. The theoretical values of ν_1/ν_2 are given in the legend along with the mass and chemical composition (X, Z). **Right:** Corresponding evolutionary tracks of the seismic models with the observed position of SX Phe.

pulsation is driven. In the case of B-type pulsators, these values depend mainly on the adopted opacity data and their modifications around the Z-bump (Daszyńska-Daszkiewicz et al. 2005a). In the case of cooler pulsators like δ Scuti or SX Phoenicis stars, the values of f are affected by convection in the envelope (Daszyńska-Daszkiewicz et al. 2003).

One aim of this paper was to obtain some constraints on the efficiency of convective transport in the outer layers of SX Phe. Semi-empirical values of f can be derived from multi-colour light variations and radial-velocity measurements (e.g., Daszyńska-Daszkiewicz et al. 2003, 2005a). To that end, we used the Strömgren amplitudes and phases for the two radial mode frequencies as determined by Rolland et al. (1991). The atmospheric flux derivatives over the effective temperature and gravity and also limb darkening were derived from the Vienna model atmospheres (NEMO, see, e.g., Nendwich et al. 2004).

Fig. 2 compares the theoretical and empirical values of f for our best seismic model (its parameters were $M = 1.05\ M_\odot$, $\log T_{\rm eff} = 3.889793$, $\log L/L_\odot = 0.84375$ and $X_0 = 0.67$ and $Z = 0.002$). The theoretical values were computed for different values of the mixing-length parameter $\alpha_{\rm MLT}$, and their empirical counterparts for different values of the microturbulent velocity, ξ_t. The left panel shows the results for the dominant frequency ν_1 (the radial fundamental mode), and the right panel for the frequency ν_2 (the first overtone radial mode).

In the case of the frequency, ν_1, there is a fairly good agreement with an MLT parameter $\alpha_{\rm MLT} \approx 0.7$ and a microturbulent velocity of $\xi_{\rm t} \approx 8$ km s^{-1}. The results for the second mode frequency, ν_2, are less unambiguous. While the imaginary part of f suggests a lower value for the MLT parameter ($\alpha_{\rm MLT} < 1.5$), the real part of it does not agree with any theoretical value. that can result from a much smaller light amplitude of ν_2, which is determined with much lower accuracy. For example, the amplitude in the Strömgren v filter is almost three times lower for ν_2 than for ν_1.

4 Conclusions

We carried out a seismic analysis of the prototype star SX Phoenicis. We started by fitting the two radial mode frequencies and constrained the mass, luminosity and chemical composition. Our best seismic model to reproduce those two frequencies has the parameters $M = 1.05\ M_\odot$, $\log T_{\rm eff} = 3.889793$, $\log L/L_\odot = 0.84375$, and a chemical composition of $X_0 = 0.67$ and $Z = 0.002$. The effective temperature was within the allowed observational range, and the luminosity agreed completely with the determination from *Gaia* DR2 data. The ages of the other seismic models which we generated were in the range 2.5–3.9 Gyr. Further studies are needed to determine the age more accurately, because it would give a clue to the star's evolutionary past and its origin.

In the next step we tried to reproduce the bolometric flux amplitude f corresponding to each mode. The aim was to get further constraints on (e.g.) convection and atmospheric conditions. The (semi)empirical values of f were derived from the Strömgren amplitudes and phases, by adopting the Vienna model atmospheres. We found that the microturbulent velocity, ξ_t, had a very strong effect on the empirical values of f; that had already

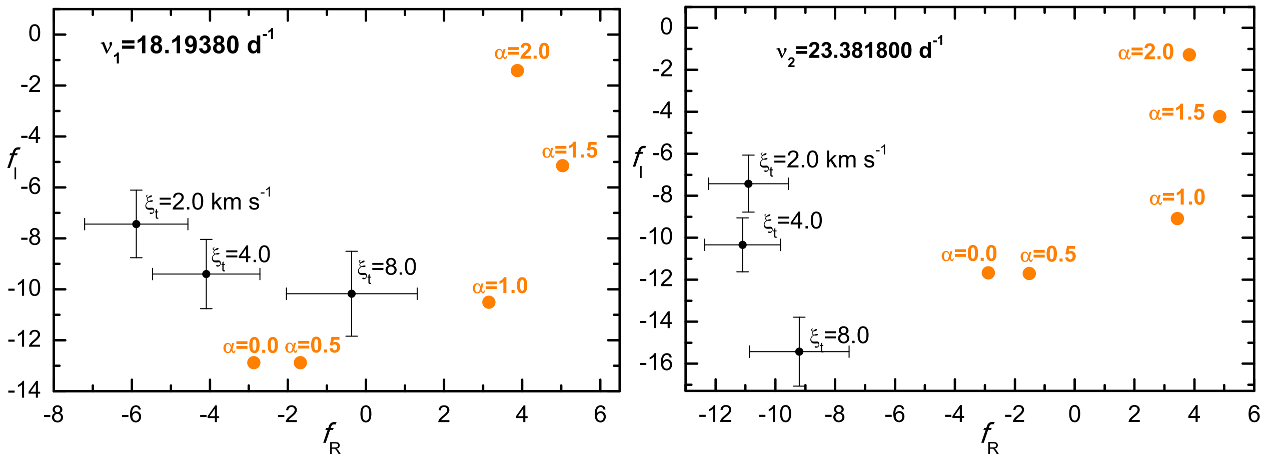

Fig. 2. Comparison of the theoretical and empirical values of f on the complex plane, for the radial fundamental mode (**left panel**) and for the first overtone mode (**right panel**). The theoretical values were computed for our best seismic model (see text) by considering different values of the mixing-length parameter $\alpha_{\rm MLT}$. Their (semi)empirical counterparts were determined from the Strömgren photometry and the Vienna model atmospheres assuming different values of the microturbulent velocity ξ_t.

been announced by Daszyńska-Daszkiewicz et al. (2005b) and Daszyńska-Daszkiewicz (2007). In the case of the fundamental radial mode, the empirical and theoretical values of f agreed if the MLT parameter was about $\alpha_{\rm MLT} = 0.7$ and the microturbulent velocities in the atmospheres were about $\xi_t = 8$ km s^{-1}. It would mean that the efficiency of convective transport in the outer layers of SX Phe is rather moderate. In the case of the first overtone mode the agreement was poor, and further studies are needed. It might have resulted from some interaction between the two modes, or a need of additional modification in pulsational and/or atmospheric modellinig. Another reason could be that smaller photometric amplitudes are determined with much lower accuracy.

We plan to extend these studies by re-computing atmospheric models for a higher helium abundance and higher microturbulent velocities. The effects of modifying the mean opacities will also be examined.

New simultaneous multi-colour photometric and spectroscopic time-series observations would definitely lead to more plausible seismic constraints on the parameters of the model and the theory.

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