

## OPEN PROBLEMS IN HIGH-MASS STELLAR EVOLUTION

S. Ekström<sup>1</sup>, G. Meynet<sup>1</sup>, C. Georgy<sup>1</sup>, R. Hirschi<sup>2</sup>, A. Maeder<sup>1</sup>, J. Groh<sup>3</sup>, P. Eggenberger<sup>1</sup> and G. Buldgen<sup>1</sup>

**Abstract.** Massive stars can roughly be divided into two categories: the one that will become red supergiants at (or shortly before) the end of their life, and those that will become Wolf-Rayet stars. RSG stars are dominated by convection, and experience a very strong mass loss; WR stars also undergo a strong mass loss, though through another process than RSGs. Both convection and mass loss don't arise naturally in 1D stellar evolution codes, so we rely on prescriptions. On top of that, most massive stars live in multiple systems, which increases even more the complexity of the picture. I will review the current status of massive star modelling, the problems we meet and some solutions that could come, either from 3D modeling, or from surveys.

Keywords: Stars: massive, Stars: evolution, Stars: mass-loss, Stars: binaries: general, Stars: rotation, Convection

### 1 Introduction

Massive stars are extreme objects that often show an extreme luminosity, bringing them close to the Eddington limit  $L_{\text{Edd}} = \frac{\kappa_{\text{es}}}{4\pi cGM}$  (where  $\kappa_{\text{es}}$  is the opacity by electron scattering, and other variables have the obvious meaning). They also experience strong mass loss, which imprints the circumstellar/interstellar medium both kinematically and chemically.

If we extend the Conti scenario for the filiation of massive stars spectral types (Conti 1975), we see that we can divide the stars in two categories: the ones that become red supergiants (RSG) at some point in their evolution, and the ones that become Wolf-Rayet stars (WR). Except in a narrow mass range between 30–40  $M_{\odot}$ , there is no overlap between the two categories.

$M > 60 M_{\odot}$ :	O → Of/WNL → LBV → WNL → WC → WO	→ SNIbc?	WR
$M = 40 - 60 M_{\odot}$ :	O → BSG → LBV → WNL → (WNE) → WC → WC → WO	→ SNIbc? → SNIbc?	
$M = 30 - 40 M_{\odot}$ :	O → BSG → RSG → WNE → WCE	→ SNIbc	
$M = 25 - 30 M_{\odot}$ :	O → (BSG) → RSG → (YSG? LBV?)	→ SNII-L/b	RSG
$M = 10 - 25 M_{\odot}$ :	O/B → RSG → (Ceph. loop for $M < 15 M_{\odot}$ ) → RSG	→ SNII-P	

Both RSG and WR are dominated by mass loss, and RSG are moreover dominated by convection, two processes that are absolutely out of reach of 1D stellar evolution codes. For the modelling of the secular evolution of stars, we are restricted to 1D codes and hence rely on prescriptions and recipes to implement these two processes. We will review below the strategies adopted in the case of convection (Sect. 2) and mass loss (Sect. 3). On top of this, two physical ingredients seem to play a major role in massive star evolution: rotation and multiplicity. We will also review the uncertainties and difficulties brought by these two ingredients (Sect. 4 and 5).

<sup>1</sup> Department of astronomy of the University of Geneva, 1290 Versoix, Switzerland

<sup>2</sup> Astrophysics group, Keele University, Keele ST5 5BG, United Kingdom

<sup>3</sup> Trinity College Dublin, The University of Dublin, Dublin, Ireland

## 2 Convection

Massive stars undergo a succession of core- and shell-burning phases, a large fraction of which are convective. The convective or radiative nature of central C-burning has been evoked as a determinant for the success of a supernova explosion (Timmes et al. 1996). Massive stars that become RSG develop an extended convective envelope. In any case, convection is a non-optional ingredient in the models. The problem we face is that convection is a highly non-1D process, being linked to turbulence.

When implementing convection in 1D codes, the first step we have to take is the definition of the convective zones. Linear perturbation theory shows that a stratified medium becomes unstable when  $\nabla_{\text{rad}} > \nabla_{\text{ad}}$  (Schwarzschild 1958) or alternatively when  $\nabla_{\text{rad}} > \nabla_{\text{ad}} + \frac{\alpha}{\delta} \nabla_{\mu}$  (Ledoux 1947), with  $\nabla_{\text{rad}} = \frac{3}{16\pi acG} \frac{\kappa LP}{MT^4}$  the radiative thermal gradient, and  $\nabla_{\text{ad}} = \frac{P\delta}{C_P \rho T}$  the adiabatic thermal gradient. Note that great care must be taken in the way this determination is done, as shown in Gabriel et al. (2014): the border must be determined from within the convective side of the boundary, not from the radiative side.

The second step is to determine the thermal gradient inside the convective zone. In the deep interior, convection can be reasonably treated as adiabatic, but near the surface, this is not the case. Most 1D codes use the mixing-length theory (Böhm-Vitense 1958, MLT), but more sophisticated, non local theories exist (Shaviv & Salpeter 1973; Maeder 1975; Roxburgh 1978; Bressan et al. 1981; Kuhfuss 1986; Langer 1986; Canuto 1992, 2011a,b,c,d,e; Xiong et al. 1997; Deng et al. 2006; Gabriel & Belkacem 2018).

There are many pieces of observational evidence that the classical Schwarzschild or Ledoux criterion underestimate the size of convective cores: measures of the width of the main sequence (MS, Maeder & Mermilliod 1981), of eclipsing binaries (Claret & Torres 2016), or asteroseismic measures (Aerts et al. 2003; Miglio et al. 2008; Moravveji et al. 2015, 2016, among others). This can be easily understood since of course, convective movements don't stop where the acceleration stops, the matter having still a velocity, as shown recently in 3D modelling (Arnett et al. 2019, see their figure 3). The region where the additional mixing occurs above a convective zone is called *overshoot*. While 3D prescriptions are on their way to help 1D modellers to implement this overshoot better, for now there are usually two different overshoot implementations in 1D stellar codes:

- **penetrative overshoot**, where the core radius is increased by a given fraction of the pressure scale height:  $R_{\text{cc}} = R_{\text{boundary}} + f_{\text{ov}} H_P$
- **exponential overshoot**, where the diffusion coefficient in the region is modified with an exponentially decaying factor:  $D_{\text{ov}} = D_{\text{conv},0} \exp\left(-\frac{2\Delta r}{f_{\text{ov}} H_P}\right)$

Both these implementations imply a calibration. There are usually two kinds of calibrations: either on the MS width, or on the drop of the surface velocity in a  $V \sin(i) - \log(g)$  diagram. In the stellar grids by Brott et al. (2011), they used the second calibration and deduced an overshoot parameter  $d_{\text{over}} = 0.3 H_P$ . In those by Ekström et al. (2012), we used the first calibration and deduced an overshoot parameter  $d_{\text{over}} = 0.1 H_P$ . The different physical inputs in the two grids can partially explain the difference found in the value to use, but it might also be linked to the region of the mass domain in which the calibration has been performed. Brott et al. (2011) have used models of  $16 M_{\odot}$ , while Ekström et al. (2012) have calibrated the MS width in the low-mass domain, where neither rotation nor stellar winds play any major role in the evolution. Several observational studies suggest that the overshoot could be mass dependent (Maeder & Mermilliod 1981; Ribas et al. 2000; Castro et al. 2014).

Comparing models computed with different codes shows large differences, especially after the main sequence (Martins & Palacios 2013; Jones et al. 2015). A large part of these differences is linked to the way convection is implemented in the various codes. Some attempts to get help from 3D modelling have been undertaken (see, for instance, Freytag et al. 1996; Arnett et al. 2015; Cristini et al. 2017). The first results show that the convective boundaries are much smoother than usually assumed in 1D codes, and that the bottom boundary is steeper than the top one (Cristini et al. 2017). Also, the simulations show that internal gravity waves are generated above the convective zone and propagate in the radiative zone, waves that could play a role in the angular momentum transport (Edelmann et al. 2019, see next section).

To go from 3D to 1D, the way is paved: we have first to identify the important parameters. A promising variable seems to be the bulk Richardson number:  $\text{Ri}_B = \frac{\Delta B \ell}{V_{\text{rms}}^2}$ , with  $\Delta B = \int_{r_c - \Delta r}^{r_c + \Delta r} (N^2 dr)$  the buoyancy jump,  $\ell$  the length scale of the fluid element, and  $V_{\text{rms}} = V_{\text{conv}} = \left(\frac{E_{\text{conv}}}{\rho}\right)^{1/3}$  the convective velocity. Then we have to test different convection regimes (shell C-, Ne-, O- burning, maybe even He), to check if they behave identically with respect of this parameter. We have to find a formulation adequate for parameters accessible in 1D codes, and finally we have to do tons of tests... It's a long process, but it is on its way, so stay tuned!

### 3 Mass loss

From the lowest mass B-type main sequence stars to the extreme luminous blue variables (LBV), the mass-loss rates encountered in massive stars cover about 8 orders of magnitude. One single star during its whole life can experience mass-loss regimes that differ up to about 4 orders of magnitude. When one observes a massive star, the mass-loss rates it is experiencing plays a major role in the stellar type determination.

Mass loss comes in two flavours: steady radiatively-driven winds, and episodic mass-loss events. Radiatively-driven winds can be fast and thin, during the main sequence, or slow and thick in the post-MS phase, while WR stars show very fast and thick winds. Episodic mass-loss events occur in the LBV phase, during the final stage instabilities, or because of binary interactions. The RSG mass loss is probably something inbetween those two flavours, resulting in a slow and thick wind, which nature is not fully understood yet.

The challenge we meet as 1D modellers is that the mass loss doesn't come from first principles in the modelling. We rely on prescriptions (see, for instance, Reimers 1975; de Jager et al. 1988; Kudritzki et al. 1987; Kudritzki & Puls 2000; Nugis & Lamers 2000; Vink et al. 2000, 2001; van Loon et al. 2005; Gräfenor & Hamann 2007, among others). 1D models need average rates, they cannot usually follow burst episodes.

The various existing prescriptions often cover only a narrow validity domain, so we have to switch from one to the other along the evolution of a given model. In case two prescriptions overlap, they often differ in their outcome and it is difficult to decide which one we should use. But even a slight change during a limited time can completely modify the endpoint and the stellar track in the Hertzsprung-Russell diagram (HRD, see Georgy & Ekström ????, Groh et al. in prep). Actually, comparisons between observations and models for massive stars are rather a check for the mass-loss prescription used than anything else.

Not only the wind prescription is a difficult choice, but also there is some uncertainties due to wind clumping. Wind clumping makes the mass-loss rates observed seem larger than they really are. It has been claimed that we should reduce the mass-loss rates by a factor of 3 to 10 (Bouret et al. 2005; Fullerton et al. 2006). However, the winds might be clumped, but it seems they might be porous as well, and neglecting the porosity of the wind could lead to underestimate the mass-loss rates (Oskinova et al. 2007; Muijres et al. 2011). So all in all, the reduction of the mass-loss rates could be only small.

### 4 Rotation

All massive stars rotate, many of them rapidly (Mokiem et al. 2006; Huang & Gies 2006; Ramírez-Agudelo et al. 2013; Simón-Díaz & Herrero 2014). The difficulty comes when we want to implement the effects of rotation in 1D codes. Some use a purely diffusive scheme, others use an advecto-diffusive scheme. The diffusive scheme is completely dependent on the choice of the  $f_C$  and  $f_\mu$  values (see Keszthelyi et al. in prep.). When the advective term is taken into account, we meet three different prescriptions for the expression of the horizontal turbulence coefficient (Zahn 1992; Maeder 2003; Mathis et al. 2004), and two different prescriptions for the expression of the shear turbulence (Maeder 1997; Talon & Zahn 1997). Moreover, Maeder et al. (2013) have shown that the different instabilities triggered by rotation cannot be taken into account separately, because they interact one with the others.

The choice of the prescriptions yields large differences in the outcome (stellar tracks in the HRD, rotation velocity evolution, chemical enrichment, etc. see Chieffi & Limongi 2013; Meynet et al. 2013).

Moreover, rotation can induce magnetic instability, which brings an additional coupling inside the star. The basic developments have been proposed by Spruit (1999, 2002). Maeder & Meynet (2004) have added the condition that the differential rotation must be large enough for triggering the magnetic instability. On the observational side, the MiMes survey have found that around 7% of OB stars present detectable magnetic fields (Grunhut et al. 2011; Wade et al. 2014, 2016). Of course, this number counts only the stars that have a surface magnetic field, and one strong enough to be detected, so it must be considered as a lower limit.

An indirect observational constraint comes from the rotation rates of white dwarfs or neutron stars. According to Suijs et al. (2008), it is clear that models without magnetic fields cannot reproduce the slow rotation of observed stellar remnants, while the inclusion of the magnetic coupling marginally helps reconcile with the observation. While for the neutron star, the explosion episode could modify the angular momentum budget one way or another, the white dwarfs experience a much smoother transition from stellar life to remnant, and hence represent strong constraints. The need for a stronger coupling has been also highlighted by asteroseismic measurements of differential rotation inside red giants (Mosser et al. 2012; Cantiello et al. 2014). Several studies have started to pave the way towards a solution (Eggenberger et al. 2012; Cantiello et al. 2014; Fuller et al. 2019; Edelmann et al. 2019). Thanks to the Kepler satellite, a handful of red subgiants have been observed and for six of them, exquisite core and surface rotation determinations were made possible. It seems from those

measurements that the coupling increases with mass but decreases with the evolution during the subgiant phase (Eggenberger et al. 2019), which is in contrast with the behaviour found for the giants, in which the coupling seems to increase with the evolutionary stage. Could this be the sign of different mechanisms at play? We definitely need more subgiant measurements before we can draw firm conclusions.

## 5 Multiplicity

Most massive stars don't live alone. Sana et al. (2012) find that 70% of O-type stars are in binaries. From the apparently single stars, only 48% of them would be real single stars, the 52% left being either merger products, or pre-, or post-interacting binaries (de Mink et al. 2014).

This fact complicates the picture in an indescribable way. Binaries are at least as complicated as two single stars, bearing the same uncertainties on convection, mass loss, rotation for each component. But binaries are more than two single stars, since they also come with additional uncertainties linked to their binary nature (Georgy & Ekström ???).

They might undergo mass-transfer episodes, being the donor or the gainer (sometimes the two situations successively during their lifetime). It is a tricky episode to model, since we do not now exactly how much of the matter released in the Roche lobe overflow should fall on the gainer and how much could be lost by the system.

Binarity complicates the rotation picture of the component, since it might induce tidal interaction, modifying the way the stars are mixed internally and the way the rotation velocity evolves (Song et al. 2016). If they undergo mass transfer episodes, their angular momentum should change accordingly, but it is not clear how to handle that aspect, what fraction of angular momentum could be really transferred, or if it could become a barrier to mass transfer.

While single star evolution can be explored with three parameters ( $M$ ,  $Z$ , and  $\Omega$ ), when it comes to binaries, an almost infinite parameter space opens, since to the three basic parameters one needs to add the separation (that can take almost any value) and mass ratio (that can be anything between 0 and 1). There are basically two different approaches to this problem. For specific cases, detailed binary models will be computed, with the simultaneous evolution of both components, and with more or less full binary physics (Cantiello et al. 2007; Eldridge et al. 2008; Eldridge & Stanway 2009). However to explore more widely the parameter space, a population synthesis is used, based often on single star models, with the help of binary prescriptions for the period and mass ratio distributions, for the mass and angular momentum transfer efficiency, or the tidal influence on rotation (de Mink et al. 2013). It must be kept in mind that stellar evolution shows often non linear behaviours, that could be completely missed by the simplifications needed to handle such a huge parameter space. Some assumptions that are usually made in these cases are sometimes questionable. For instance, immediate synchronisation is assumed most of the time, while in models computed with tidal mixing and all the other internal mixing processes, the result is that synchronisation is achieved only temporarily, but soon the natural evolution of the star overcomes this forcing and the velocity of the star drops below the orbital angular velocity (Song et al. 2016). Careful observations of the velocity of the two components of a binary also shows that synchronisation might not be the rule (Martins et al. 2017; Putkuri et al. 2018). Another assumption is that during the common envelope phase, the envelope is completely ejected. 3D simulations do not show this, with only about 10% of the envelope ejected very early in the spiral-in phase (Ricker & Taam 2008, 2012; Passy et al. 2012; Ohlmann et al. 2016). Of course, 3D simulations do not cover long timescales, so it might be that the envelope is ejected on longer, thermal timescales. Also most of the simulations do not take into account the recombination processes, which could play a role in the ejection (Nandez et al. 2015). However this shows that some caution is still needed when pondering the results of binary population synthesis.

## 6 Conclusions

Our understanding of massive star evolution still faces many open problems. Some of them concern very basic physical inputs (convection, mass loss), some concern more sophisticated (but essential) modelling ingredients (rotation  $\pm$  magnetic fields, multiplicity).

Some help can come from 3D modelling, studying short phases or parts of stars. It could in particular give us prescriptions for the treatment of convection, the inclusion of internal gravity waves, and help us understand what occurs in weird phases of binary evolution. Unfortunately, the computational cost of multi-D modelling prevents us from using them to study the secular evolution of stars.

To make further progress, we need large surveys with exquisite precision and consistent analysis. Unfortunately this is seldom compatible: either we benefit from large survey results, or we benefit from detailed analysis of observational data on a handful of stars.

I take the opportunity here to address a caveat to stellar models users: be careful when mixing models from different codes. We have just seen that the differences in the physical inputs are large and have important consequences on the outcome. Mixing models could however be used to get some sorts of theoretical error bars on the model's predictions.

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