Stars and their variability observed from space
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CONVECTION IN ROTATING STARS: CONVECTIVE PENETRATION AND MIXING

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Abstract. This poster examined a model for rotating convection in stars and planets. The convection model is used as a boundary condition for a first-order expansion of the equations of motion in the transition region between convectively unstable and stably-stratified regions, estimating the depth of convective penetration into the stable region and establishing a relationship between that depth and the local convective Rossby number. Several models for the mixing in such a region were considered.

Keywords: stars: convection, rotation, mixing

1 Introduction

Convective flows cause mixing not only in regions of superadiabatic temperature gradients but in neighboring subadiabatic regions as well, as motions from the convective region contain sufficient inertia to extend into those regions before being braked buoyantly or eroded turbulently (e.g., Miesch 2005; Lecoanet et al. 2015; Viallet et al. 2015). This convective penetration and turbulence can thus alter the chemical composition and thermodynamic properties in those regions (e.g., Zahn 1991; Augustson et al. 2016; Pratt et al. 2017a). One such convection model has been described in Paper I (Augustson & Mathis 2019). The motivation for taking rotation into account in this model was the numerical work of Käpylä et al. (2005) and Barker et al. (2014), who found that the rotational scaling of the amplitude of the temperature, its gradient, and the velocity field compared well with those derived by Stevenson (1979). Moreover, the analysis by Howard (1963) showed that a principle of heat-flux maximization provided a sound basis for the description of Rayleigh–Bénard convection, triggering its use here. Thus, two hypotheses underlie the convection model: the Malkus conjecture that convection arranges itself to maximize the heat flux, and that the nonlinear velocity field can be characterised by the dispersion relationship of the linearised dynamics. Constructing the model of rotating convection then consists of three steps: first, to derive a dispersion relationship that links the normalized growth rate $\hat{s} = s/N_*$ to $q = N_{*,0}/N_*$, which is the ratio of superadiabaticity of the nonrotating case to that of the rotating case, where $N_*^2 = |g\alpha_T\beta|$ is the absolute value of the square of the Brunt-Väisälä frequency, g is the magnitude of the gravity, α_T is the coefficient of thermal expansion, and β is the temperature gradient. The next is to apply the normalized wavevector $\xi^3 = k^2/k_z^2$ (where $k_z = \pi/l$, with l being the depth of the convective layer) to maximize the heat flux with respect to ξ. The last is to assume an invariant maximum heat flux that closes this three-variable system.

To that end, a local region was considered, as in Paper I, where a small 3D section of the spherical geometry was the focus of the analysis. As shown in figure 1 of that Paper, this region covers a portion of both convectively stable and unstable zones, where the setup is configured for a low-mass star with an external convective envelope. Those regions may be exchanged when considering an early-type star with a convective core. In this local frame, there is an angle between the effective gravity $\mathbf{g}_{\rm eff}$ and the local rotation vector that is equivalent to the co-latitude, θ. The Cartesian coordinates are defined such that the vertical direction z is anti-aligned with the gravity vector, the horizontal direction y lies in the meridional plane and points toward the north pole defined by the rotation vector, and the horizontal direction x is equivalent to the azimuthal direction. The assumption of this convection model is that the magnitude of the velocity is defined as the ratio of the maximizing growth rate and wavevector. With that approximation, the velocity amplitude can be defined relative to the case of nondiffusive and nonrotating scales without a loss of generality, as

$$\frac{\mathrm{v}}{\mathrm{v}_0} = \frac{k_0}{s_0}\frac{s}{k} = \frac{5}{\sqrt{6}}\frac{N_*}{N_{*,0}}\frac{\hat{s}}{\xi^{3/2}} = \frac{5}{\sqrt{6}}\frac{\hat{s}}{q\xi^{3/2}} = \left(\frac{5}{2}\right)^{\frac{1}{6}}\xi^{-\frac{1}{2}}. \tag{1.1}$$

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Only the maximising wavevector therefore needs to be determined in order to ascertain the relative velocity amplitude. From all those equations, the horizontal wavevector may be seen to be roots of the fourteenth-order polynomial:

$$\begin{aligned}&\xi^3\left(V_0\xi^2+\tilde{s}\right)^2\left[3V_0K_0\xi^4\left(2\xi^3-3\right)+\tilde{s}\xi^2(V_0+K_0)\left(4\xi^3-7\right)+\tilde{s}^2\left(2\xi^3-5\right)\right]\\&-\frac{6\cos^2\theta}{25\pi^2\mathrm{Ro}_\mathrm{c}^2}\left[2\tilde{s}\left(K_0-V_0\right)+3\tilde{s}^2\xi+\tilde{s}\left(K_0+5V_0\right)\xi^3+3K_0V_0\xi^5\right]=0,,\end{aligned}\tag{1.2}$$

where V_0 and K_0 are the non-dimensional diffusivities and $\tilde{s}$ is a numerical factor. To ascertain the maximising wavenumber, and thus the velocity, of the motions that maximise the heat flux, one therefore supplies the co-latitude, θ, and the convective Rossby number of the flow, Ro_c.

Once the quantities relating to the convection model have been defined, the impact of rotation on the convective penetration can be characterised. Following Paper I, we determined that the depth of convective penetration scaled as $L_P/L_{P,0} = (\mathrm{v}/\mathrm{v}_0)^{3/2}$. Then, considering the extreme-value models of penetration developed by Pratt et al. (2017b), one can improve the initial estimate of Baraffe et al. (2017). Using the above extension of the Zahn (1991) model, one could then estimate both the penetration depth and the level of diffusive turbulent mixing. Taking the parameters of the Gumbel distribution as in Pratt et al. (2017a) yields the following description of the radial dependence of the diffusion: coefficient

$$D_\mathrm{v}\left(r\right)=\left(\frac{5}{2}\right)^{\frac{1}{6}}\frac{\alpha H_{P,0}\mathrm{v}_0 h}{3\sqrt{z}}\left\{1-\exp[-\exp\left(\left(r-r_c\right)/\lambda L_P+\mu/\lambda\right)]\right\},,\tag{1.3}$$

where r_c is the base of the convection zone and $\mu = 5\times10^{-3}$ and $\lambda = 6\times10^{-3}$ are the empirically determined parameters from Baraffe et al. (2017). Likewise, following the analysis of Korre et al. (2019), one can find that

$$D_\mathrm{v}\left(r\right)=\left(\frac{5}{2}\right)^{\frac{1}{6}}\frac{\alpha H_{P,0}\mathrm{v}_0 h}{3\sqrt{z}}\exp\left(-\frac{\left(r-r_c\right)^2}{2\delta_G^2}\right),\quad\text{where}\quad\delta_G=1.2\frac{\mathrm{v}}{\mathrm{v}_0}\left(\frac{E_0P_r}{SRa_0}\right)^{1/2},,\tag{1.4}$$

where S is the stiffness of the stable interface, Ra_0 is the Rayleigh number, P_r is the Prandtl number, and E_0 is the energy in the nonrotating convection. These models have now to be implemented in stellar evolution codes (Michielsen et al. 2019), and to be assessed with seismic constraints and the observed surface abundances.

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