

## STELLAR INCLINATION ANGLES FROM BE STAR $H\alpha$ EMISSION-LINE PROFILES

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### Abstract.

Radiative transfer modelling of the  $H\alpha$  emission-line profile of a Be star can be used to extract a reliable estimate of the angle between the observer's line-of-sight and the central B star's rotation axis, commonly called the stellar inclination angle. We verify this radiative transfer approach by comparing two inclination angles derived geometrically for a sample of eleven Be stars with interferometric observations that resolve the star's disk light distribution on the sky. With this  $H\alpha$  profile-fitting technique, we can use Be stars as probes in young open clusters to detect potential correlations between the orientation of stellar spins.

Keywords: Stars: emission-line, Be, rotation, open clusters and associations: general

### 1 Introduction

Stellar rotation axes are usually assumed to be randomly oriented in space, and in that case a distant observer will measure inclination angles  $i$  (the angle between the line-of-sight and the stellar rotation axis) that satisfy a  $\sin i$  distribution Gray (1992). As it is difficult to measure the angle  $i$  for any individual star, the  $\sin i$  distribution is usually assumed to be valid and is often used to derive other quantities, such as the distribution of stellar equatorial speeds. Nevertheless, most (perhaps all) OB stars are born in clusters, originating from the gravitational collapse of the parent molecular cloud (Lamb et al. 2010). The details of this collapse, and the interplay between gravity, rotation, turbulence, and magnetic fields, ultimately sets the angular momentum acquired by individual stars. Given the complexity of this process, it is not *a priori* obvious that the individual stellar spins will be randomized. Numerical simulations (Rey-Raposo & Read 2018) suggest that strongly-aligned stellar spins can result if more than  $\approx 40\%$  of the initial kinetic energy of the parent cloud is in the form of rotation.

There are several possible approaches to determining the inclination angles of individual stars in order to test the assumption of random orientations; a measurement of the projected equatorial rotational speed ( $v \sin i$ ) from the broadening of spectral lines plus an estimate of the star's underlying rotation period from photometric or magnetic variations can be used to extract  $i$  (Abt 2001, for example.). Very rapid stellar rotation leads to gravitational darkening in which the star becomes oblate, with a temperature variation over its surface (Collins 1965); as the star's spectrum then depends on how it is viewed, an estimate of  $i$  can be extracted from carefully modelling its spectrum (Zorec et al. 2016, and references within). Another interesting approach is the use of asteroseismology (Gizon & Solanki 2003). Using this method, Corsaro et al. (2017) have recently detected the first observational evidence of strong spin alignment between red giants in two old Galactic open clusters, NGC 6791 and NGC 6819, based on three years of *Kepler* data. Nevertheless, all three of these methods are data intensive and require extensive time series and/or high signal-to-noise observations. In this paper we propose a simpler method of overcoming these limitations based on the spectra of Be stars.

### 2 Method

A Be star is a rapidly-rotating, B-type, main-sequence star surrounded by a circumstellar disk. The most widely accepted disk formation model invokes episodes of near critical rotation of the central B star, in which material is ejected from the stellar equator into a disk (Granada et al. 2013). The disk then spreads outward by the action of turbulent viscosity, forming a viscous decretion disk (Rivinius et al. 2013). The spectra of Be stars

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show emission in the hydrogen Balmer series, most notably  $H\alpha$ . It has long been known that the morphology of the  $H\alpha$  emission line reflects the inclination angle of the central B-type star (Porter & Rivinius 2003), from singly-peaked emission for  $i \approx 0^\circ$  (pole-on star and face-on disk), to double-peaked emission for intermediate  $i$ , to doubly-peaked emission with strong central (shell) absorption for  $i \approx 90^\circ$  (equator-on star and edge-on disk). This suggests that detailed profile fitting of the  $H\alpha$  emission line based on radiative transfer modelling may be able to extract a reliable measure of the stellar inclination angle.

We have used the **Beray** radiative transfer code (Sigut 2011, 2018) to compute theoretical  $H\alpha$  profiles for a large number of Be-star models. The thermal structure of the circumstellar disk was determined using the **Bedisk** code (Sigut & Jones 2007) which enforces radiative equilibrium in a gas of solar chemical composition heated by the photoionizing radiation of the central B star. The disk density was taken to fall with radius as a power-law of index  $n$ , starting from a value  $\rho_0$  at the stellar surface. For each spectral type of the central B-type star, the  $H\alpha$  line profile is a function of four model parameters: the base disk density,  $\rho_0$  (range  $10^{-12}$ – $10^{-10}$   $\text{gm cm}^{-3}$ ), the power-law index,  $n$  (range 1.5–4.0), the outer disk radius,  $R_d$  (range 5–65  $R_\odot$ ), and the system viewing inclination,  $i$  (range  $0^\circ$ – $90^\circ$ ). Models were computed for eleven spectral types between B0 and B9, making a total  $H\alpha$  library of over 231,000 profiles.

To extract the inclination angle for a Be star, its observed  $H\alpha$  line profile is compared to model profiles in the line library of the appropriate spectral type. The comparison uses a figure-of-merit, taken to be the absolute percentage difference between the observed profile and the model profile, and the disk parameters  $(\rho_0, n, R_d, i)$  of the profile that minimize the figure-of-merit are adopted as representing the star best. One potential issue is that there may be degeneracies among the four model parameters such that a wide range of model inclinations  $i$  may fit the observed profile equally well by adjusting the values of the other parameters  $(\rho_0, n, R_d)$ . We tested this issue extensively via Monte Carlo simulations, in which simulated observed  $H\alpha$  profiles were generated with  $S/N = 10^2$  and resolution  $\mathcal{R} = \lambda/\Delta(\lambda) = 10^4$  (both typical of Be star observations) and fitted them to the  $H\alpha$  profile libraries. In examining the distributions of the  $(\rho_0, n, R_d)$  parameters corresponding to the best 15% of fits to each simulated profile, we found that the inclination angle was robustly recovered, typically to within  $\pm 10^\circ$ . This issue is discussed further in the following section, where we use observed, and not simulated, profiles.

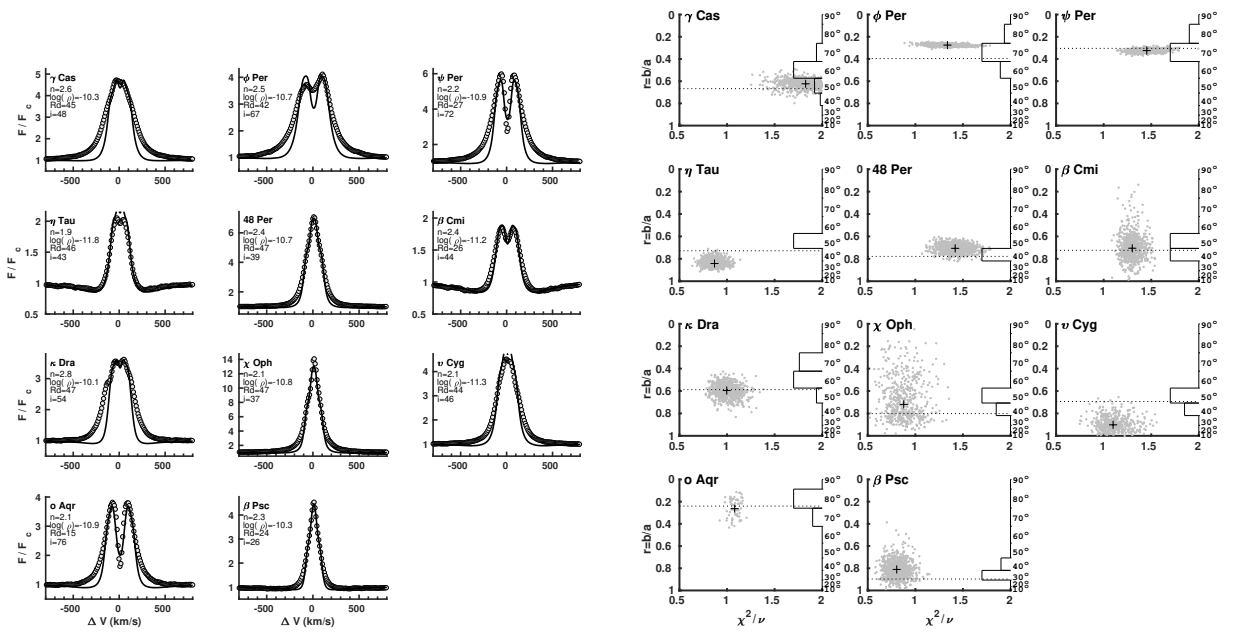
### 3 Comparison with interferometric observations

To test how the  $H\alpha$  profile-fitting method performs, we assembled a sample of eleven Be stars with published interferometric visibilities from the Naval Precision Optical Interferometer (Armstrong et al. 1998). Interferometric observations can resolve the Be star disk on the sky, and purely geometric models can be fitted to the light distribution. A particularly simple and successful model is an elliptical Gaussian disk (Tycner et al. 2003). In it, four parameters are fitted to the observed visibilities: a major axis ( $a$ ), a minor axis ( $b$ ), a position angle on the sky ( $\chi$ ), and a brightness ratio between the disk and the central star ( $c_*$ ). As Be star disks are equatorial and circular, the observed axial ratio is simply due to projection through the stellar inclination angle, i.e.  $r \equiv b/a = \cos i$ ; hence, the system inclination can be recovered geometrically from the interferometric observations in a manner that is independent of any radiative transfer modelling. Of course, the simple relation  $r = \cos i$  must fail for sufficiently large  $i$ , as the disk’s minor axis will be limited by its finite scale height; however, simulated images computed with **Beray** suggest that this only occurs for  $i > 80^\circ$ .

The following eleven Be stars (with references to the available interferometry) form our sample:  $\gamma$  Cas (Tycner et al. 2003);  $\eta$  Tau,  $\beta$  CMi (Tycner et al. 2005);  $\phi$  Per (Tycner et al. 2006);  $\kappa$  Dra,  $v$  Cyg,  $\beta$  Psc (Jones et al. 2008);  $\chi$  Oph (Tycner et al. 2008),  $o$  Aqr (Sigut et al. 2015), 48 Per (Jones et al. 2017), and  $\phi$  Per (Sigut et al. 2019, private communication). Observed  $H\alpha$  line profiles were taken from observations at the John S. Hall telescope at Lowell Observatory; all spectra have  $S/N = 10^2$  and  $\mathcal{R} = 10^4$ .

Fits to all eleven observed  $H\alpha$  line profiles, along with the best-fitting disk parameters and inclinations, are shown in the left panel of Fig. 1. In most cases the fits are good; however, in some cases ( $\phi$  Per, for example) there is clearly a systematic difference between the available models and the observations. In addition, the profiles in this sample are nearly symmetric, whereas many Be stars show asymmetric profiles with  $V/R$  variations, i.e. sizable differences between the height of the red (R) and blue (V) emission peaks (Steff et al. 2009). In tests, we have had success in fitting the red and blue sides of asymmetric  $H\alpha$  profiles separately, extracting a consistent  $i$  from both fits; however, this is outside the scope of the current project.

The right panel of Fig. 1 shows the fits to the observed interferometric visibilities presented as axial ratios  $r \equiv b/a$  as a function of the reduced  $\chi^2$  of the fit. The cloud of points for each star represents 500 bootstrap Monte Carlo re-samplings of the interferometric data, and the cloud of points gives a visual representation of the uncertainties. Also shown in each stellar panel is the histogram of model inclinations for all library profiles that



**Fig. 1. Left:**  $H\alpha$  library profile fits to the sample of Be stars. The observed  $H\alpha$  profile for each star is shown by the circles, and the best-fitting model profile by a solid line. The adopted disk parameters and inclination are indicated for each star. **Right:** Interferometric axial ratios  $r = b/a$  as a function of the reduced  $\chi^2$  of the fit for the sample Be stars. The cross in each panel is at the median axial ratio and fit reduced  $\chi^2$ . Histograms on the right of each stellar panel give the distribution of inclinations of all model  $H\alpha$  profiles that have a fit with a figure-of-merit within 15% of the best-fitting profile.

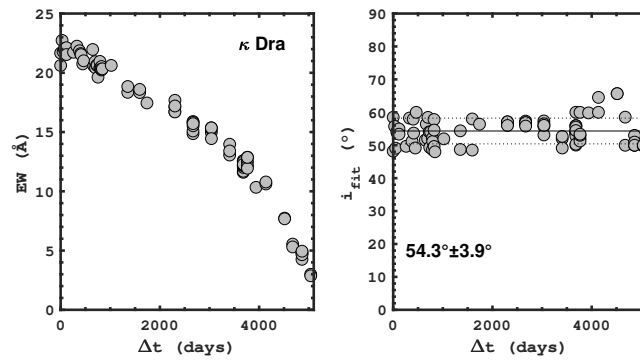
fitted the observed  $H\alpha$  line profile with a figure-of-merit within 15% of the best-fitting profile. These histograms show visually the degeneracy in the spectroscopic viewing inclination over the other model parameters. The horizontal dotted line in each panel is a “refined” inclination found via interpolation in the library models. In all of these panels, axial ratios and inclination angles have been related using  $r = \cos i$ . The difference between the spectroscopic inclinations and interferometric inclinations is consistent with a normal distribution of mean  $\mu = -1^\circ$  and a standard deviation of  $\sigma = 7^\circ$ . Thus, based on this sample, stellar inclinations of an accuracy of  $\approx \pm 10^\circ$  are obtainable from  $H\alpha$  line-profile fitting.

Another important characteristic of Be stars is their variability (Porter & Rivinius 2003). Be star disks are often observed to form and dissipate on time-scales of several years to decades, as set by the disk’s viscous time-scale. An example of disk dissipation is shown in Fig. 2 for the star  $\kappa$  Dra (HD 109387); over nearly 14 years of observations, its  $H\alpha$  profile shows a factor of 5 decrease in emission strength. We have fitted all 85  $H\alpha$  profiles in this time-series using the methods outlined above to extract the system inclination at each epoch – which we expect to be unchanging, despite the disappearance of the disk. As shown in Fig. 2, all profiles are consistent with a system inclination of  $i = 54^\circ \pm 4^\circ$ , independent of the strength of the  $H\alpha$  emission.

## 4 Conclusions

We have demonstrated that an observation of the  $H\alpha$  emission-line profile of a single Be star (with moderate signal-to-noise and resolution) is sufficient to constrain the inclination angle of the central B-type star to within  $\approx \pm 10^\circ$ . As Be stars comprise upwards of one-fifth of all main-sequence B stars (Zorec & Briot 1997), Be stars provide a promising avenue to look for possible spin-correlations between massive stars in young open clusters.

Finally, we note another intriguing possibility. The stellar inclination angle  $i$  constrains the star’s rotation axis to lie along a cone of opening angle  $i$  around the line of sight. However, Be stars offer the possibility to obtain the position angle of the rotation axis within this cone via polarization measurements. The continua of Be stars are weakly polarized owing to electron scattering from the circumstellar disk. The position angle of the polarization vector is perpendicular to the scattering plane, i.e., the disk, and in the direction of the stellar rotation axis. Hence,  $H\alpha$  profile fitting combined with polarization measurements for Be stars will enable the reconstruction of the 3D orientation of the central B star’s rotational axis.



**Fig. 2. Left:** H $\alpha$  equivalent widths as a function of time (days since first observation) for the Be star  $\kappa$  Dra. A positive equivalent width indicates emission. The initial observation was on 2005 March 3. **Right:** Recovered stellar inclination for the central B star of  $\kappa$  Dra as a function of time, based on H $\alpha$  line profile fitting. The solid line is the mean inclination; the dotted lines give the  $1\sigma$  variation.

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